Incomplete Financial Markets and Differential Information

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Abstract. We provide a model of an incomplete markets economy where private restrictions on consumption are interpreted as lack of information. The existence of an equilibrium where agents are unable to infer any additional information from prices is proved

When assets are nominal, these non-enlightening equilibrium prices lead us to a natural selection of equilibria where the degree of real indeterminacy is reduced. Finally, we present an example illustrating that consumption constraints can remove completely the real indeterminacy of equilibria.

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1 Introduction

We provide a model of incomplete financial markets with differential information. In order to do this, we first consider an incomplete markets economy with two periods and uncertainty in the second period, where trading is restricted not only by a financial structure but also by restrictions on individual consumption sets. The restrictions for an individual consumer are given by a partition of the set of states of nature, with the interpretation that this consumer limits her choice to one commodity bundle from each element of the partition. Under standard assumptions, we prove the existence of equilibrium and also obtain differentiability properties of demand functions. Next, we describe our incomplete markets economy as an economy with differential information. This is achieved by reconsidering the restrictions on consumption sets as private information structures [1]. Thus our model allows us to provide a differential information approach to incomplete markets economies with financial assets and to deal with several issues regarding prices and equilibrium outcomes. Indeed, for the differential information interpretation of our economy, we are led to seek existence of an equilibrium in which prices do not reveal information. Further, as we will show for the case of nominal assets, an asymmetric information framework gives insights into the degree of real indeterminacy of equilibria.

We recall that in the Arrow-Debreu model all commodities are traded simultaneously, no matter when or under what state of nature they are consumed, and then each consumer faces only one budget constraint. In contrast, the distinguishing feature of our model with incomplete markets is that consumers face a multiplicity of budget constraints; in the first period, before the state of nature is known, agents transact securities and trade commodities, in the second period, after the realization of the state of nature, transactions occur in commodities. Thus we cannot avoid in our differential information economy the possibility of making inferences about the state of nature taking into account market prices. In other words, when the asset market is incomplete, in conjunction with the financial structures, prices determine the attainable reallocations of revenue and, therefore, the role of prices extends beyond conveying the scarcity of commodities. Precisely, we stress that a differential information approach within an

[1]Within a complete markets framework, the literature on differential information defines private information sets in a manner similar to the way we define consumption restrictions (see, for instance, Radner, 1968, and Yannelis, 1991).

incomplete markets setting leads to a situation where prices may also convey information across individuals. This diversity of the roles of prices affects the existence, optimality and indeterminacy of equilibria.

In order to be consistent with the informational approach, we consider an equilibrium concept where prices do not provide any additional information to agents. We remark that these non-enlightening prices, which are compatible with the common information, are precisely those that entail the private restrictions on consumption. This is so because, when these non-revealing prices prevail, the maximization of preferences leads to allocations in accordance with the private information structures.

Further, when individuals select their consumption plans and their portfolios, not only prices may transmit information but also market aggregate variables will reflect, to a greater or lesser extent, the accumulate information perceived. Even when prices are restricted to those which are non-enlightening or, equivalently, compatible with the common information structure, the total excess of demand of commodities may transmit some information which goes beyond the underlying common private information. Therefore, to obtain existence of a non-informational equilibrium we require the compatibility of the aggregate excess demands for commodities with respect to the common information. In order to reach a better understanding of this additional assumption, which guarantees the existence of equilibrium prices that do not reveal additional information to any consumer, we state conditions on the individual information structures, on the endowments and on preferences across states ensuring that for every price the aggregate excess of demand is measurable with respect to the common information. Moreover, when there is only one commodity to be traded in each state, then the excess of demand does no longer reveal information.

Thus we state the existence of non-informational equilibrium for the economy with numeraire assets and private information structures, basically assuming that the parameters defining the economy do not provide information to any consumer. Furthermore, the proof provided of the equilibrium existence allows us to conclude that this existence result still holds if we consider assets which pay in money, that is, nominal assets.

We remark that the existence result of the non-enlightening equilibrium obtained differs from other results in the literature in certain respects, essentially because the way we introduce private information within an incomplete market economy is different from previous approaches. In this paper the structure of private information is fixed and, therefore, agents sign contracts that are compatible with their private information. Thus we do not consider a signal space which involves a set of realizations of private information, since in our economy private information is just a restriction on consumption, and so we define information following the seminal paper by Radner (1968). It is important to note that prices do not reveal any private information ex ante, which is in sharp contrast with the rational expectations equilibrium model (see Radner, 1979).

Following rational expectations models as in Radner (1979) (instead of Radner, 1968), Polemarchakis and Siconolfi (1993) and Rahi (1995) introduce information into incomplete markets economies by considering a space of signals and studying the existence of non-revealing equilibrium. Further, Polemarchakis and Siconolfi (1993) prove that non-informative rational expectations equilibrium exists by exploiting the real indeterminacy of equilibria. More recently, Boisdeffre and Cornet (2002) consider the same signal space considered by Rahi (1995) and deal with the issue of arbitrage with differential information and incomplete financial markets, with a focus on the information that no-arbitrage asset prices can reveal. Our existence result is substantially different since, as we have already remarked, the equilibrium solutions differ mainly in the considerations regarding the definition of information structures. Moreover, the existence result that we show does not rely on the indeterminacy of equilibria and we do not deal with the issue of arbitrage with differential information. Instead, we prove the existence of non-informational equilibrium for numeraire assets and our proof can also be used to show the existence of equilibrium for the case of nominal assets.

Concerning nominal assets, our aim is to analyze the degree of real indeterminacy in relation with the private information structures that we introduce in economies with incomplete markets. It is known that when assets pay in money the indeterminacy of equilibria is not only nominal, as in the case of complete markets, but has real implications (see Balasko and Cass, 1989, and Geanakoplos and Mas-Colell, 1989). The basic economic reasons for this real indeterminacy come from the fact that different price levels, across states of nature, lead to different purchasing power of the nominal asset returns across states that in turn, when the financial markets are incomplete, result in changes in the income transfers achievable by trading assets.

The differential information framework induces us to explore the generic occurrence of a continuum of equilibrium allocations and to analyze in greater depth economic approaches to reduce the degree of real indeterminacy[2]. Thus this paper provide a substantially new approach to the problem of real indeterminacy of equilibria in financial models.

Related papers that address the real indeterminacy of the equilibrium problem, put forward several approaches to deepen the model structure by providing different procedures to study such real indeterminacy. Both Pesendorfer (1995) and Bisin (1998) state models where the asset structure is endogenously given and the consideration of intermediation costs becomes the main driving force behind the reduction of real indeterminacy. In contrast, Magill and Quinzii (1992) as well as Faias, Moreno-García and Páscoa (2002) consider asset structures exogenously given, although the ways to overcome the indeterminacy problem differ; in the former the price of money across states is endogenously determined whereas in the latter the relative rates of inflation are endogenously determined by agents that become monopolists of certain commodities.

In this paper, we state a model where assets are exogenously given, but we consider neither a monetary framework nor market power of agents as the source to prevent indeterminacy. We are led to handle the problem of indeterminacy by using a procedure different from those contemplated in the existing literature; namely, we consider a differential information framework as the driving force to reduce indeterminacy in a model with exogenous assets.

The existence result of non-enlightening equilibrium prices in models with nominal assets allows us to define selections of equilibria and go further by enhancing the analysis. Precisely, the equilibrium selection is obtained by restricting prices to those which do not reveal any information. This equilibrium selection diminishes the real indeterminacy degree and permits to attain an interesting conclusion from the existence of non-informational equilibrium prices. Actually, it is important to remark that equilibria with non-enlightening prices have the property that the purchasing power of the units of account is constant for states which are not distinguished by all the consumers and can only vary across common information events. Furthermore, the differences on private and individual information structures are the main cause of the possible variations

[2] *Degree of indeterminacy* means the highest dimension of a manifold contained in the set of equilibrium allocations of commodities.

of the purchasing power of the units of account that can be observed at the selection of equilibria that we suggest. Indeed, the reduction of the degree of real indeterminacy depends crucially on the fact that, at the non-enlightening equilibrium prices, the inflation rates may become different just across common knowledge events and not across all the states.

Finally, at the end of the paper, we present an example which illustrates that consumption constraints by themselves not only reduce but completely remove the indeterminacy of equilibria.

The remainder of the paper is organized as follows. Section 2 formalizes the model of an incomplete market economy with private restrictions on consumption and states an equilibrium existence result and differentiability properties of the demand functions. In Section 3 we present a differential information approach to financial markets and show the existence of equilibrium when prices do not reveal information. In Section 4 we consider an economy with nominal assets and define selections of equilibria for which the degree of real indeterminacy is reduced. Section 5 contains an example pointing out how consumption restrictions can even remove the real indeterminacy of equilibria. A final Appendix presents the proofs of all the results stated in this paper.

2 A Model with Financial Assets and Consumption Restrictions: Existence of Equilibrium

In this Section, we establish a model of incomplete markets, with two periods and uncertainty in the second period, where assets pay in units of a numeraire (say the first physical commodity in each state of nature) and consumers privately require specific conditions on consumption. Hence, the main difference between our economy and the usual general equilibrium model with real numeraire assets is that consumption sets are given by individual and private requirements and, therefore, may differ among agents.

The economy $\mathcal{E}$ evolves over two periods $(t = 0, 1)$ with S possible states of nature $(s = 1, \dots, S)$ in the second period. There are L physical commodities $(\ell = 1, \dots, L)$ available in $t = 0$. At date $t = 1$, in each state, L spot markets open, where the L consumption commodities are traded.

There are N consumers $(i = 1, \dots, N)$ in the economy. Each consumer i is characterized by a preference relation on $\mathbb{R}_+^{L(S+1)} \times \mathbb{R}_+^{L(S+1)}$ represented by the utility function U^i and an initial endowment vector $\omega^i \in \mathbb{R}_+^{L(S+1)}$ with $\omega^i = (\omega_0^i, \omega^i(s), s = 1, \ldots, S)$ where ω_0^i denotes the initial endowment in $t = 0$ and $\omega^i(s)$ denotes the initial endowment in $t = 1$ for the state s. Moreover, each individual i is also characterized by a restricted consumption set $\mathcal{X}_i \subset \mathbb{R}_+^{L(S+1)}$ which is described by means of a partition $\mathbb{P}_i$ of the set of states as follows.

Let $\mathcal{S}$ denote the finite set of possible states of nature and let $\mathcal{P}$ denote the set of partitions of $\mathcal{S}$. The private restrictions on consumption plans are given by elements of $\mathcal{P}$. For each $s \in \mathcal{S}$ let $\mathbb{P}(s)$ denote the event in the partition $\mathbb{P}$ that contains s. We will refer to a function with domain $\mathcal{S}$ which is constant on each event of $\mathbb{P}$ as $\mathbb{P}$-*measurable*, although, strictly speaking, measurability is with respect to the σ-algebra generated by the partition. A partition defines restrictions on consumption by requiring the commodity bundles to be constant within each event in such a partition. That is, given $\mathbb{P}_i$, the consumption set of agent i is $\mathcal{X}_i = \{(x_0, x(s), s \in \mathcal{S}) \in \mathbb{R}_+^{L(S+1)} \mid (x(s), s \in \mathcal{S})$ is $\mathbb{P}_i$-measurable$\}$. We remark that the consumption set $\mathcal{X}_i$ depends on an additional consumer's characteristic which is a private requirement of each agent.

In short, in this economy, each consumer i is characterized not only by preferences and resources (U^i, ω^i) but also by a private consumption set $\mathcal{X}_i$, which actually restricts her consumption plans by requiring the same commodity bundle in states belonging to the same event in accordance with an associated private partition $\mathbb{P}_i$ of the set of states.

Consumers can transfer wealth across states by trading in the financial markets in the first period. There is a finite number of real numeraire assets in the economy, indexed by $b = 1, \ldots, B$. Each asset b promises to deliver $R^b(s) \in \mathbb{R}_+$ units of the first commodity, in each state of nature $s = 1, \ldots, S$ at the second period. Let $R = (R^b(s))_{s=1,\ldots,S}^{b=1,\ldots,B}$ denote the $S \times B$ payoff matrix describing the financial market structure of returns in this economy $\mathcal{E}$. In order to address an incomplete markets framework, we assume that $B < S$. We also assume that R has full rank (i.e., the rank of R is B).

At date $t = 0$, agents take portfolio decisions and make consumption plans for the different possible states taking into account their private characteristics. That is, agents trade the B assets and the L physical commodities at date 0 and trade the L physical commodities at date 1 according to their restrictions on consumption.

Therefore, the incomplete financial markets economy $\mathcal{E}$, where agents face different consumption sets given by individual private requirements, is described by

$$\mathcal{E} \equiv (R, \mathcal{X}_i, U^i, \omega^i, i = 1, \ldots, N)$$

An allocation $x = (x^i, i = 1, \ldots, N) \in \mathbb{R}_+^{N(S+1)L}$ of commodities is *feasible* in the economy $\mathcal{E}$ if $x^i \in \mathcal{X}_i$ for every i, $\sum_{i=1}^N x_0^i \le \sum_{i=1}^N \omega_0^i$ and $\sum_{i=1}^N x^i(s) \le \sum_{i=1}^N \omega^i(s)$ for every state $s \in \mathcal{S}$. A portfolio or asset allocation $y = (y^i, i = 1, \ldots, N) \in \mathbb{R}^{BN}$ is feasible if $\sum_{i=1}^N y^i = 0$.

Let $p_0 = (p_\ell)_{\ell=1}^L$ denote the price vector of commodities at time $t = 0$ and let $p(s) = (p_\ell(s))_{\ell=1}^L$ denote the spot price vector of commodities at time $t = 1$ in the state s. Then, $p_1(s)$ denotes the price of the designated numeraire commodity at state s. Let $p = (p_0, p(1), \ldots, p(S))$ denote the vector of commodity prices. Finally, the asset prices vector is denoted by $q \in \mathbb{R}^B$. We recall that q is a non-arbitrage price if and only if there is no $y \in \mathbb{R}^B$ with $q \cdot y = 0$ and $Ry > 0$. In this paper, in order to obtain well-defined demand functions, we consider only those asset prices which satisfy the non-arbitrage condition.

Given a price system $(p, q) \in \mathbb{R}_+^{L(S+1)} \times \mathbb{R}_+^B$, the budget set for agent i is given by

$$\begin{aligned} B^i(p,q) = \{(x,y) \mid \quad & x \in \mathcal{X}_i,\ p_0 \cdot (x_0 - \omega_0^i) + q \cdot y \le 0 \text{ and} \\ & p(s) \cdot (x(s) - \omega^i(s)) \le p_1(s) R(s) \cdot y, \text{ for every } s \in \mathcal{S}\}. \end{aligned}$$

Each consumer i behaves as price-taker and, given (p, q), maximizes U^i on $B^i(p, q)$. Note that, this individual problem can be equivalently formalized by considering the utility function U^i restricted to $\mathcal{X}_i$.

Now, we define the concept of equilibrium in our economy.

Definition 2.1 *An equilibrium for the economy $\mathcal{E}$, with differentiated and restricted consumption sets, is a price system (p, q) and a feasible allocation of commodities and portfolios (x, y) such that*

(i) every agent i maximizes U^i on the budget constraint $B^i(p, q)$ and

(ii) $p_0 \cdot \sum_{i=1}^N (x_0^i - \omega_0^i) + \sum_{s=1}^S p(s) \cdot \sum_{i=1}^N (x^i(s) - \omega^i(s)) = 0.$

We emphasize that condition (i) requires, in particular, that equilibrium allocations belong to the restricted consumption sets. Condition (ii) is implied by the Walras law and ensures that, in equilibrium, if a commodity is in excess of supply its price is zero.

In the remainder of this Section, we state existence of equilibrium for our economy and we also show differentiability properties of the individual demand functions. In order to facilitate the reading of this paper, all the proofs are relegated to a final Appendix.

In order to obtain equilibrium existence, we state the following assumptions:

(A.1) For every i, the utility function $U^i : \mathbb{R}_+^{L(S+1)} \to \mathbb{R}$ is continuous, increasing and quasi-concave.

(A.2) For every i, $\omega^i \gg 0$ and $w^i \in \mathcal{X}_i$.

(A.3) Given any state s there exists an agent $i \in N$ such that, $\{s\} \in \mathbb{P}_i$.

Assumption $(A.1)$ and positivity of endowments are standard hypotheses in the related literature. Assumption $(A.2)$ requires also that endowments are consistent with the private consumption restrictions. Further, assumption $(A.3)$ ensures that given any state there is at least one consumer who allows free trade (i.e., with no private consumption restriction) in such a state. This hypothesis allows us to prove existence of equilibria with nonnegative prices and with no free disposal. Note that, whenever there exists an agent whose consumption set is $\mathbb{R}_+^{L(S+1)}$, assumption $(A.3)$ holds.

Observe that, since the budget constraints present a degree zero homogeneity with respect to the first period prices, we can choose $(p_0, q) \in \Delta^{L+B-1}$ where $\Delta^{L+B-1} = \{(p_0, q) \in \mathbb{R}_+^{L+B} : \sum_{l=1}^{L} p_{0l} + \sum_{j=1}^{B} q_j = 1\}$.

To prove the existence of equilibrium we first show the continuity property of the budget correspondences for prices $(p_0, q) \in \Delta^{L+B-1}$ and prices $p(s)$, for each state s which guarantee the existence of interior points for the budget sets. Precisely, the next Lemma will be used in the proof of the main result in this Section which states equilibrium existence.

Lemma 2.1 *For every consumer i the budget correspondence B^i takes non-empty and convex values and is continuous at every prices such that $(p_0, q) \in \Delta^{L+B-1}$ and $p(s)\omega^i(s) > 0$, for every s.*

Theorem 2.1 *Let $\mathcal{E}$ be an economy under assumptions (A.1)-(A.3). Then there exists an equilibrium $(p, q), (x, y)$; such that every price is strictly positive, $(p_0, q) \in \Delta^{L+B-1}$, $p(s) \in \Delta^{L-1}$ for every $s \in \mathcal{S}$ and there is no free disposal.*

We remark that if we drop assumption $(A.3)$ then we can not ensure market clearing at equilibrium. To illustrate this point, we state the following example of an economy, where preferences are strictly monotone and the consumption restrictions become the source of free disposal at equilibrium allocations. Consider an incomplete market economy with two agents 1 and 2, three states of nature $\{a, b, c\}$, and one commodity in each state. The endowments are $\omega^1 = (5, 5, 0)$ and $\omega^2 = (5, 0, 5)$. Both agents have the same preference relation which is represented by the utility function $U(x_a, x_b, x_c) = x_a^{1/2} + x_b^{1/2} + x_c^{1/2}$. The consumption sets are given by $\mathbb{P}_1 = \{\{a, b\}, \{c\}\}$ and $\mathbb{P}_2 = \{\{a, c\}, \{b\}\}$, respectively. There are two assets with returns $R^1 = (0, 1, 0)$ and $R^2 = (0, 0, 1)$. It can be shown that the collection $q = (1/2, 1/2), p_a = 0, p_b = p_c = 1,\ y^1 = (-1, 1), y^2 = (1, -1), x^1 = (4, 4, 1), x^2 = (4, 1, 4)$ is an equilibrium with free disposal. Actually, there is not market clearing in state a provided that $\{a\}$ belongs neither to $\mathbb{P}_1$ nor to $\mathbb{P}_2$.

Finally, we state differentiability properties of the demand functions. For it, we assume that R is in general position which means that every submatrix of R has full rank.

Proposition 2.1 *Let R be in general position. Assume that agent i allows free trade in at least $S - B$ states and the utility function U^i is twice continuously differentiable, strictly increasing and concave. Then, the individual demand of agent i is a C^1 function.*

3 Equilibria with Non-enlightening Prices

In this Section, first we recast the private consumption restriction previously stated as individual information structures. This informational approach to incomplete markets leads us to define a notion of equilibrium where prices reveal no information. Then, we show existence of non-informational equilibrium.

Following the literature on differential information economies [3], let us interpret the set of events $\mathbb{P}_i$, which previously defined consumption restrictions, as a private information structure for agent i. Thus, in this paper, let $\mathcal{E}_I$ denote an incomplete market economy with differential information which is described by $(R, \mathbb{P}_i, U^i, \omega^i, i = 1, \ldots, N)$, where $\mathbb{P}_i$ is a partition of the set of states of nature representing the individual and private information for each consumer i.

In the previous Section, we have proved the existence of equilibrium for an economy $\mathcal{E}$ with consumption restrictions where prices have no informative role. In contrast, if we establish information considerations in the incomplete markets setting, prices may convey information across individuals. Actually, when the information privately available differs among agents, prices acquire an informational role in addiction to that of conveying the aggregate scarcity of commodities. In order to be consistent with the informational approach, we will consider an equilibrium concept where prices do not reveal additional information to any agent.

For this, let $\mathbb{P}$ denote the associated common information, which is the meet of the partitions $(\mathbb{P}_i, i = 1, \ldots, N)$, and we write $\mathbb{P} = \bigwedge_{i=1}^{N} \mathbb{P}_i$ [4]. Then, E is said to be a common information event if $\mathbb{P}_i(s) \subset E$ for every state $s \in E$ and for every agent i. Note that $\{s\} \in \mathbb{P}$ if and only if $\mathbb{P}_i(s) = \{s\}$ for every i or, equivalently, information does not lead to any consumption restriction at the state s for any agent.

Next, we precise an equilibrium solution defined by prices that are compatible with the common information and, then, do not reveal any information to consumers.

Definition 3.1 *A non-informational equilibrium for the economy $\mathcal{E}_I$ with differential information is a price system (p, q) and a feasible allocation of commodities and portfolios (x, y) such that*

(i) the commodity price system p is $\mathbb{P}$-measurable,

[3] In this literature, independently of the role of prices, an agent is not able to distinguish among states belonging to the same event included in her information set and therefore the consumption is required to be the same in the states that the agent is not able to discern.

[4] The meet is the largest σ-algebra which is contained in each σ-algebra generated by $\mathbb{P}_i$, for every i. That is, $\mathbb{P}$ is the finest partition of the set of states that is coarser than each $\mathbb{P}_i$.

(ii) every agent i maximizes U^i on the budget constraint $B^i(p,q)$ and

(iii) $p_0 \cdot \sum_{i=1}^{N}(x_0^i - \omega_0^i) + \sum_{s=1}^{S} p_s \cdot \sum_{i=1}^{N}(x_s^i - \omega_s^i) = 0.$

We remark that in the economy $\mathcal{E}_I$ consumers make economic decisions (financial and consumption plans choices) based on diverse pieces of information. In fact, market aggregate variables may refine the individual information sets. Actually, by observing a market aggregate, each agent may be able to infer something about the private information possessed by others. In particular, even when prices are restricted to those with a non-enlightening property regarding individual and private information, the total excess of demand for commodities may transmit an information which goes beyond the underlying common information structure. Therefore, when we attempt to decentralize consumer behavior by means of a price market system which conveys no information, it becomes natural to assume that neither the asset returns nor the behavior of agents reveal any information. Thus, in order to get existence of equilibrium with non-enlightening prices we state the following assumptions:

(A.4) The matrix R is $\mathbb{P}$-measurable.

(A.5) Given a price system $p \gg 0$ which is $\mathbb{P}$-measurable and asset prices q, with $0 \ll (p_0, q) \in \Delta^{L+B-1}$, $\sum_{i=1}^{N}(x^i(p,q) - \omega^i)$ is $\mathbb{P}$-measurable, where $x^i(p,q)$ denote the demand of commodities of agent i at prices (p,q).

Assumption $(A.4)$ requires the matrix R to be measurable with the common information structure and, therefore, the return matrix does not convey additional information to agents. Assumption $(A.5)$ requires the aggregate excess of demand to be constant among common information events. This implies that when observing this aggregate economic variable no agent can refine the partition which defines her private information.

Theorem 3.1 *Let $\mathcal{E}_I$ be an incomplete markets economy with differential information under assumptions* $(A.1) - (A.5)$. *Then, the set of non-informational equilibria is non-empty.*

Remark 1. We remark that assumption (A.5) can not be deleted in the above existence result. To show this, consider an economy with two agents, 1 and 2, two states, a and b, two commodities, x and y, in $t = 0$ and in each state. There is one asset with return matrix $R = (1, 1)$. Both agents have the same preferences represented by the utility $U(x_0, y_0, x_a, y_a, x_b, y_b) = x_0 y_0 + x_a y_a + x_b y_b$. The private information structures are given by $\mathbb{P}_1 = \{\{a\}, \{b\}\}$ and $\mathbb{P}_2 = \{a, b\}$, respectively. The endowments are $\omega_0^1 = \omega_b^1 = \omega_0^2 = \omega_a^2 = \omega_b^2 = (1, 1)$, whereas $\omega_a^1 = (1, 2)$. In this economy the assumption (A.5) does not hold and there is no equilibrium price which is non-revealing.

Remark 2. Let us consider the particular case where utility functions are both separable and measurable with respect the private information structures; further, the utilities corresponding to each states are strictly quasi-concave. That is, for each agent i, $U^i(x) = V^i(x_o) + \sum_{s \in \mathcal{S}} u^i(s, x_s)$, where $u^i(s, \cdot)$ is strictly quasi-concave for every s and $u^i(s, \cdot) = u^i(s', \cdot)$ for every $s' \in \mathbb{P}_i(s)$. If p is a price system which is $\mathbb{P}_i$-measurable, then it is easy to show that $x^i(p, q)$ is $\mathbb{P}_i$-measurable and, hence, we can drop the informational restrictions in the problem for agent i. This implies that if we restrict prices to be non-revealing (i.e., to be $\mathbb{P}$-measurable), the individual optimization behavior guarantees that allocations will be compatible with the private information structures. That is, when non-enlightening prices prevail, the maximization of preferences leads to allocations in accordance with the informational requirements. Therefore, we conclude that prices which are compatible with the common information actually leads to allocations which are informationally feasible without requiring such a condition explicitly. In this sense, we can consider the non-informational equilibria as those in which the prices ensure that for every consumer i the commodity bundles $x^i \in \mathcal{X}_i$ for any equilibrium allocation $x = (x^i, i \in N)$.

The remainder of this section is devoted to explore two economic settings where assumption $(A.5)$ holds and therefore we obtain existence of equilibrium with non-enlightening prices. The first one addresses economies with just one commodity in each state whereas the second one is related to a condition on the states inside a common information that we call *anonymity property* of states.

3.1 Economies with one commodity in every state

Consider the particular case of an economy $\mathcal{E}_I$ with just one commodity in each state. If the return matrix R is $\mathbb{P}$-measurable (that is, assumption $(A.4)$ holds) then the assumption $(A.5)$ holds as well. To show this, let $p \gg 0$ be

$\mathbb{P}$-measurable price system and $q \gg 0$ a vector of asset prices. Since the matrix R is $\mathbb{P}$-measurable we have that, for every s, s' belonging to the same common information event, the following equalities hold:

$$p(s)\sum_{i=1}^{N}(x_s^i(p,q) - w_s^i) = p(s)R(s)\cdot\sum_{i=1}^{N} y^i =$$
$$= p(s')R(s')\cdot\sum_{i=1}^{N} y^i = p(s')\sum_{i=1}^{N}(x_{s'}^i(p,q) - w_{s'}^i).$$

As there is just one commodity in each state, $p(s) \in \mathbb{R}_+$, for every s. Therefore, the aggregate excess demand is $\mathbb{P}$-measurable. Finally, we conclude that when there is only one commodity in every state assumption $(A.5)$ is implied by $(A.4)$.

3.2 Common information events with anonymous states

We say that an economy satisfies an anonymous states criterion whenever, by observing the behavior of individuals, it is not possible to identify different states which belong to the same common information event. In other words, states included in a common information event are undistinguishable by an external agent that perceives the aggregate consumption decisions of agents.

Thus, an economy presents common information events with anonymous states when the next requirement holds: if there is a consumer making economic decisions in the states s, then there is also another consumer who behaves exactly in the same way in any other state which belongs to the same common information event. We remark that when this requirements holds the aggregate excess demand is measurable with respect the common information, i.e., assumption $(A.5)$ holds.

To illustrate this, let $\mathcal{E}_I^A$ be an economy with nN consumers indexed by ih. Each consumer ih is characterized by the information $\mathbb{P}_{ih}$, the initial endowments ω^{ih} and the utility function U^{ih}. Assume that $\mathbb{P}_{ih} = \mathbb{P}_i$ and $\omega_0^{ih} = \omega_0^i$ for every $h = 1, \ldots, n$. Let $\mathbb{P} = \{E_1, \ldots, E_C\}$ denote the common information structure. Note that each private information defines a partition of every common knowledge event. That is, given an event $E_e \in \mathbb{P}$, for each agent i, we can write $E_e = \bigcup_{k=1}^{n_e^i} P_{e,k}^i$ with $P_{e,k}^i \in \mathbb{P}_i$ such that $P_{e,k}^i \bigcap P_{e,k'}^i = \emptyset$ for every $k \neq k'$. Assume that $n_e^i = n$ for every i and e. Let us fix a representative state of nature $s_{e,k}^i$ in each element $P_{e,k}^i$.

To define the endowments and utilities for consumers ih we state some notations. For each h let π^h denote the following permutations of elements belonging to the index set $\{1, \ldots n\}$:

$$\pi^h(k) = \begin{cases} k+h-1 & \text{if } k+h-1 \leq n \\ k+h-n-1 & \text{otherwise} \end{cases}$$

Note that if a plan belongs to $\mathcal{X}_i$, it suffices to define x in the first period and in the representative elements of each event. Let us consider the function f^h which associates to each $x \in \mathcal{X}_i$ the consumption plan $\hat{x} \in \mathcal{X}_i$ given by $\hat{x}_0 = x_0$ and $\hat{x}(s^i_{e,k}) = x(s^i_{e,\pi^h(k)})$.

For every ih the initial endowments are $\mathbb{P}_i$-measurable and are given by $\omega^{ih}(s^i_{e,k}) = \omega^i(s^i_{e,\pi^h(k)})$. Finally, the utility function U^{ih} is given by $U^{ih}(x) = U^i(f^h(x))$ for any $x \in \mathcal{X}_i$. Note that $\omega^{i1} = \omega^i$ and $U^{i1} = U^i$ for every $i = 1, \ldots, N$.

We remark that the total initial endowments in the first period are equal to $n \sum_{i=1}^N \omega^i_0$ whereas the total initial endowments in the second period in any state $s \in E_e$ are given by $\sum_{i=1}^N \sum_{k=1}^n \omega^i(s^i_{e,k})$. Therefore, total initial endowments in the economy $\mathcal{E}^A_I$ are $\mathbb{P}$-measurable.

Let $x^{ih}(p,q)$ and $y^{ih}(p,q)$ denote the demands of agent ih for commodities and assets. Assume that R is $\mathbb{P}$-measurable and let p be a price system for commodities which is $\mathbb{P}$-measurable as well. It is not hard to show that $x^{i1}(p,q)(s^i_{e,k}) = x^{ih}(p,q)(s^i_{e,\pi^h(k)})$ and $y^{i1}(p,q) = y^{ih}(p,q)$ for every $h = 1, \ldots, n$. We conclude that in the economy $\mathcal{E}^A_I$ the set of states which form a common information event satisfies the anonymity property above and, as in the case of one commodity, assumption $(A.5)$ is implied by $(A.4)$.

4 Information and Real Indeterminacy

Consider now that assets are nominal. It is known that, in economies with assets that pay in units of accounts, the indeterminacy of equilibria is not only generically nominal but has also real implications. In fact, in incomplete markets models, with nominal assets and S possible states of nature in the second period, there are $S+1$ Walras law and only two independent sources of homogeneity in demand. Precisely, in a two period general equilibrium securities model, when assets pay in money, the generic dimension of the set of equilibrium allocations, in the incomplete markets situation, is $S-1$ and, therefore, the indeterminacy depends only on the number of states (see Balasko and Cass, 1989, and Geanakoplos and Mas-Colell, 1989).

In this Section, we show how the private consumptions constraints, rewritten as differentiated information structures, give light to propose selections of equilibria with the aim of reducing the degree of real indeterminacy. For this, let $\mathcal{E}_n$ be an incomplete markets economy with differential information, as the economy $\mathcal{E}_I$ described previously, except that the B assets in $\mathcal{E}_n$ are nominal instead of being numeraire. Note that given prices for the first commodity in every state of nature, the model with nominal assets becomes identical to a model with numeraire assets (after dividing the nominal returns by the fixed prices). In other words, given $\lambda = (\lambda_1, \ldots, \lambda_S) \in \mathbb{R}^S_+$, the economy $\mathcal{E}_n$ with nominal assets can be interpreted as an economy $\mathcal{E}_\lambda$ with numeraire assets (being commodity 1 the numeraire in every state) in which the return matrix is defined by $\Lambda^{-1}(\lambda)R$, where $\Lambda(\cdot)$ is an operator that maps a vector $\lambda \in \mathbb{R}^S_+$ into a diagonal matrix $\Lambda(\lambda)$ of order $S \times S$ whose diagonal elements are $\lambda_1, \ldots, \lambda_S$.

Therefore, Theorem 3.1 shows existence of non-informational equilibrium not only for the economy $\mathcal{E}_I$, with real numeraire assets, but also for the nominal assets economy $\mathcal{E}_n$. That is, if the economy $\mathcal{E}_n$ with nominal assets and differential information satisfies assumption (A.1)-(A.5), then the set of non-informational equilibria is non-empty. This existence result for equilibria with non-enlightening prices allows us to define a selection of equilibria for differential information economies with nominal assets. In which follows we will show, this selection, that is based on information issues, reduces the degree of real indeterminacy of equilibrium. In fact, the presence of differences on information across agents, which basically means a restriction on the individual consumption sets, reduces the possibilities of trade resulting in a reduction of the degree or real indeterminacy when equilibria are restricted to the selection provided by the differential information approach.

Recall that the basic reasons for real indeterminacy of equilibria in a model with nominal assets can be summarized as follows: nominal assets are contracts which promises returns in units of account; variations in the purchasing power of the unit of account across states result in different equilibria and the standard model does not present any endogenous mechanism which determines such a purchasing power across states. Thus, we find that any change in the relative rates of inflation across the states has a real effect, even if it is perfectly anticipated.

By considering only those equilibrium prices which do not reveal information, we are thoroughly selecting those equilibria where the purchasing power of the units of account can no longer vary across states which belong to the same common information event (i.e., states that are not distinguished by all the agents) and can only differ across common information events. In other words, the equilibrium selection we propose may present different inflation rates across common knowledge events but prevents inflation rates to modify within common knowledge events. This property of the non-informational equilibria allows us to show that the degree of real indeterminacy of equilibria is reduced. In this way, we suggest a mechanism, which is based on a differential information approach, that selects just some precise alterations in the purchasing power of the unit of account. Precisely, we state the following result.

Theorem 4.1 *Let $\mathcal{E}_n$ be an incomplete markets economy with nominal assets and differential information under assumptions* $(A.1)-(A.5)$ *and let* C *be the cardinality of the partition which defines the common information structure. Then, the degree of real indeterminacy for the non-informational selection of equilibria is at most $C-1$.*

Observe that when $C = 1$ there is no event different from the whole set of states which is differentiated by all the agents. If it is the case, there is just one common information event and by considering the non-informational selection, the real indeterminacy of equilibria is completely removed. On the other hand, when every agent has fully information in the economy $\mathcal{E}_n$, i.e., $C = S$, then every equilibrium is trivially non-enlightening and therefore there is no reduction of the indeterminacy degree.

The Theorem above shows that the degree of real indeterminacy is at most $C-1$. In which follows we state conditions under which generically there are $C-1$ dimensions of real indeterminacy. For it, given the return matrix R under assumption (A.4), let R_C denote the matrix of order $C \times B$ that is formed by the C rows corresponding to each common information event. Note that R_C is obtained from R by keeping just one row for each common knowledge event and then removing the repeated rows that are associated to states belonging to the same event. On the other hand, let N_C be the number of consumers whose information structure coincides with the common information structure.

In order to strengthen the result that has been previously presented in Theorem 4.1, we state the following assumptions:

(A.1)' For every consumer i, the closure of the indifference curves of U^i do not intersect the boundary of $\mathbb{R}_+^{L(S+1)}$. Furthermore, every utility function U^i is strictly increasing, twice differentiable and concave on $\mathbb{R}_{++}^{L(S+1)}$.

(A.4)' R is $\mathbb{P}$-measurable, $O < B < C$ and R_C is in general position.

In order to obtain a well-defined continuously differentiable excess demand function, Assumption (A.1)' joint with the full rank requirement for the matrix R have been required in Geanakoplos and Polemarchakis (1986) and Geanakoplos and Mas-Colell (1989) . Here, both Assumptions (A.1)' and (A.4)' joint with some requirements on N_C will allow us to show the adequate properties of the excess of demand function which are necessary for transversality arguments to be applied.

Theorem 4.2 *Let $\mathcal{E}_n$ be an incomplete markets economy with nominal assets and differential information under assumptions $(A.1)', (A.2), (A.3), (A.4)'$ and $(A.5)$. The following statements hold:*

(i) If $N > B$ and $N_C > B$ then, generically, there are $C - 1$ dimensions of real indeterminacy for the non-informational selection of equilibria.

(ii) Furthermore, if $N > CB$ and $N_C > CB$ then the entire set of non-informational equilibrium allocations is a C^1 manifold with dimension $C - 1$.

We remark that the equilibrium selection provided in this paper, which relies on differential information considerations within an incomplete markets model, differs from other approaches that have already been considered in previous papers also with the aim of going into the problem of real indeterminacy of equilibria in greater depth. Magill and Quinzii (1992) examine a general equilibrium model with money and nominal assets, where spot price levels are essentially determined by the quantity of money in the economy, and show how introducing money eliminates real indeterminacy. Pesendorfer (1995) develops a model with perfectly competitive financial innovators who design derivatives on existing nominal securities and adjust their payoffs to inflation. Pesendorfer then shows that real indeterminacy decreases when transaction costs tend to zero since then financial markets approach completeness. Bisin (1998) considers a model of financial innovation, based on intermediation costs and strategic interaction across intermediaries, where payoffs are fully indexed on the rates of inflation across states and, therefore, the indeterminacy is completely removed even for arbitrarily high costs. More recently, Faias, Moreno-García and Páscoa (2002) propose a selection mechanism by suggesting that there is one equilibrium that prevails over the others, as a result of the market power of agents that become monopolist of certain commodities in some states of nature. Here, we consider neither a monetary framework nor strategic behavior of agents as the source to prevent indeterminacy. Instead, we consider a differential information approach as the driving force behind reducing indeterminacy in a model with exogenous assets.

5 An Example

In this Section, we state an example which illustrates that restrictions on consumption, as those considered in the general model of Section 2, may also remove by themselves the real indeterminacy of equilibria without requiring a non-informational selection.

Consider an economy with two agents, two states of nature, one commodity in each state and one asset with returns $R(1) = R(2) = 1$. As in the example stated by Bisin (1998) both agents have the same quasi-linear preference relation represented by the utility function $U(x) = x_0 - 1/2\sum_{s=1}^{2}(H - x(s))^2$. The initial endowments are $\omega^1 = (2H, 1, 1)$ for agent 1 and $\omega^2 = (4, 2, 4)$ for agent 2. It can be checked that an equilibrium is given by $p_0 = p_1 = p_2 = 1,\ q = 2H - 4,$ $y^1 = 1,\ x^1 = (4, 2, 2), x^2 = (2H, 1, 3)$. We remark that, given a positive real number λ, if we take the return matrix $R_\lambda(1) = \lambda,\ R_\lambda(2) = 1$ we obtain a different equilibrium allocation $(x^1_\lambda, x^2_\lambda)$ which is also an equilibrium allocation for the initial economy. Then, we have a real indeterminacy of equilibria and, as it is known, the degree of indeterminacy is one. We remark that in the equilibria associated with $\lambda \neq 1$ the consumption for agent one differs between states. Consider now that agent 1 requires the same consumption in both states whereas agent 2 has no restrictions on consumption. In this case, there are just two equilibrium allocations which are given by $x^1 = (4, 2, 2), x^2 = (2H, 1, 3)$ (corresponding to $\lambda = 1$) and the initial endowment allocation (corresponding to $\lambda \neq 1$), respectively.

We remark that, in this example, the consumption restriction of agent 1 prevents rates of inflation to entail real effects and therefore reduces the possibilities of trade. In fact, when the rate of inflation between state 1 and state 2 are not equal to 1, there is no transaction of the asset.

Appendix

Proof of Lemma 2.1. Since the allocation $(\omega^i, 0)$ belongs to $B^i(p,q)$ for every (p,q), the budget correspondence has non-empty values and, by definition, $B^i(p,q)$ is convex for every (p,q).

Let (p^n, q^n) be a sequence of prices converging to (p,q) and let (x^n, y^n) be a sequence such that $(x^n, y^n) \in B^i(p^n, q^n)$ and (x^n, y^n) converges to (x,y). Since x^{ni} is $\mathbb{P}_i$-measurable for all n, we have that x^i is $\mathbb{P}_i$-measurable and, then, we conclude that the allocation (x,y) belongs to $B^i(p,q)$. Therefore, the correspondence B^i is upper-hemicontinuous.

Now, let us show that the interior of $B^i(p,q)$ is non-empty for every commodity price system p such that $p(s) \cdot \omega^i(s) > 0$ for every s. If $p_0 \cdot \omega_0^i > 0$, then the null bundle $x = 0$ together with a portfolio y such that $q \cdot y < p_0 \cdot \omega_0^i$ and $p_1(s)(R(s) \cdot y) \geq 0, s = 1, ..., S$ belongs to the interior of $B^i(p,q)$. Otherwise, $p_0 \cdot \omega_0^i = 0$. This implies that q is a non-zero price vector and then we can take y such that $q \cdot y < 0$ and $p(s) \cdot \omega^i(s) + p_1(s)(R(s) \cdot y) > 0$ for every state s. Therefore, we conclude that there is an interior point (x,y) in $B^i(p,q)$. Take a sequence (x^n, y^n) that converges to (x,y) and such that $x^n \in \mathcal{X}_i$ for every n. Then if (p^n, q^n) is a sequence of prices converging to (p,q), we have that for n large enough $p_0^n \cdot (x_0^n - \omega_0^i) + q^n \cdot y^n < 0$ and $p^n(s) \cdot (x^n(s) - \omega^i(s)) < p_1(s)(R(s) \cdot y^n)$. That is, (x^n, y^n) belongs to the interior of $B^i(p^n, q^n)$ for n large enough, which implies that the correspondence given by the interior of the budget correspondence is lower-hemicontinuous at (p,q). Since the closure of a lower-hemicontinuous correspondence is also lower-hemicontinuous the lower-hemicontinuity of B^i follows.

Q.E.D.

Proof of Theorem 2.1. Given the economy $\mathcal{E}$ and a compact set $K \subset \mathbb{R}_+^{L(S+1)} \times \mathbb{R}^B$, with $(\omega, 0) \in K$, consider the generalized game played by the N consumers and by $S+1$ auctioneers (one auctioneer for the first period and one auctioneer for each state of nature of the second period). Each consumer i maximizes U^i on the budget set $B^i(p,q) \bigcap K$. The first period auctioneer chooses prices $(p_0, q) \in \Delta^{L+B-1}$ and maximizes $p_0 \cdot \sum_{i=1}^{N}(x_0^i - \omega_0^i) + q \cdot \sum_{i=1}^{N} y^i$. Each second period auctioneer, in state s, chooses prices $p(s) \in \Delta^{L-1}$ and maximizes $p(s) \cdot \sum_{i=1}^{N}(x^i(s) - w^i(s))$.

The strategy sets for the auctioneers, Δ^{L+B-1} and Δ^{L-1}, respectively, are non-empty compact and convex. The payoff functions of the auctioneers are linear and then concave on their strategy and continuous on the strategy profile. By assumption (A.1), the payoff of consumers are quasi-concave and continuous and by Lemma 2.1, the budget correspondences of the consumers have non-empty and convex values and are continuous. Moreover $B^i(p,q)\bigcap K$ is compact. Therefore, there exists an equilibrium for the generalized game.

Let (x,y,p,q) be a strategic equilibrium. The behavior of the $S+1$ auctioneers guarantees $\sum_{i=1}^{N} y^i \leq 0$, $\sum_{i=1}^{N}(x_0^i - w_0^i) \leq 0$ and $\sum_{i=1}^{N}(x^i(s) - w^i(s)) \leq 0$, for every s.

To see that financial markets clear, assume that $R \cdot \sum_{i=1}^{N} y^i < 0$. Then, by the non-arbitrage condition, we have $q \cdot \sum_{i=1}^{N} y^i < 0$, implying that $p_0 \cdot \sum_{i=1}^{N}(x_0^i - w_0^i) > 0$, which is a contradiction. Then, $R \cdot \sum_{i=1}^{N} y^i = 0$ and, provided that R has rank B, we conclude $\sum_{i=1}^{N} y^i = 0$.

Now, consider a sequence of increasing compact sets K^n. For each n, let (x^n, y^n, p^n, q^n) be an equilibrium for the game restricted to K^n. Given that, $\sum_{i=1}^{N} x^{ni} \leq \sum_{i=1}^{N} \omega^i$ and $(p^n, q^n) \in \Delta^{L+B-1} \times (\Delta^{L-1})^S$ there exists a converging subsequence of (x^n, p^n, q^n) with limit $(\tilde{x}, \tilde{p}, \tilde{q})$. By both monotonicity of preferences and assumption (A.3), we can deduce that all budget constraints hold with equality. Suppose that $\tilde{p}_1(s) > 0$ for all s. Then, by inversion of a Cramer subsystem of budget equations, for each consumer i and each n large enough, we can write $y^{ni} = \frac{1}{p_1^n(s)}\ \tilde{R}^{-1}[p^n(s) \cdot (x^{ni}(s) - \omega^i(s))]_{s\in\beta}$, where $\tilde{R}$ is a non-singular submatrix of R, and β is the respective set of row indices. To see that it is impossible that $p_1^n(s)$ converges to 0 for some s, assume that there exists s such that $\tilde{p}_1(s) = 0$. By assumption (A.3) there exists a consumer j such that $\{s\} \in \mathbb{P}_j$. Let z^n be the commodity bundle which coincides with $(1 - p_1^n(s))x^{nj}$ except for commodity 1 at state s that is given by $z_1^n(s) = (1 - p_1^n(s))x^{nj} + \min_{\ell\in L} \omega_\ell^j(s)$. Note that z^n is $\mathbb{P}_j$-measurable and $((1 - p_1^n(s))y^j, z^n) \in B^j(q^n, p^n)\bigcap K^n$ for n large enough. By monotonicity and continuity of preferences, $U^j(z^n) > U^j(x^{nj})$ for every n large enough, which is a contradiction.

Now, since (x^n, p^n) converges to $(\tilde{x}, \tilde{p})$, we obtain that y^n converges to $\tilde{y}$ being

$\tilde{y}^i = \frac{1}{p_1(s)}\ \tilde{R}^{-1}[\tilde{p}(s) \cdot (\tilde{x}^i(s) - \omega^i(s))]_{s\in\beta}$.

Let us show that $(\tilde{x}, \tilde{y}, \tilde{p}, \tilde{q})$ is an equilibrium for the economy $\mathcal{E}$. It is easy to check that $(\tilde{p}_0, \tilde{q}) \in \Delta^{L+B-1}$, $\tilde{p}(s) \in \Delta^{L-1}$, for every s; $\sum_{i=1}^{N} \tilde{x}^i \leq \sum_{i=1}^{N} \omega^i$ and $\sum_{i=1}^{N} \tilde{y}^i = 0$. By monotonicity of preferences we also have that $\sum_{i=1}^{N}(\tilde{x}_0^i - \omega_0^i) = 0$ and $\tilde{p}_0 \gg 0$. Further, $\tilde{x}^i$ is $\mathbb{P}_i$-measurable, $(\tilde{x}^i, \tilde{y}^i) \in B^i(\tilde{p}, \tilde{q})$ for every agent i, and $(\tilde{x}, \tilde{y})$ is feasible. Assume that $(\tilde{x}^i, \tilde{y}^i)$ is not an optimal choice for consumer i at prices $(\tilde{p}, \tilde{q})$. Then, there exists $(\hat{x}^i, \hat{y}^i)$ in the interior of the budget set $B^i(\tilde{p}, \tilde{q})$ such that $U^i(\hat{x}^i) > U^i(\tilde{x}^i)$. For n large enough and λ sufficiently close to zero we have $z = \lambda(\hat{x}^i, \hat{y}^i) + (1 - \lambda)(\tilde{x}^i, \tilde{y}^i)$ belonging to $K^n \bigcap B^i(p^n, q^n)$. By convexity and continuity of preferences, $U^i(z) > U^i(x^{ni})$ for n large enough. This is a contradiction with the fact that (x^n, y^n, p^n, q^n) is an equilibrium for all n.

To finish the proof, let us show that there is no free disposal. Suppose that $\sum_{i=1}^{N} \tilde{x}_\ell^i(s) < \sum_{i=1}^{N} \omega_\ell^i(s)$ for a state of nature s and for a physical commodity ℓ. This implies $\tilde{p}_\ell(s) = 0$. By assumption (A.3), there exists an agent j for which $\mathbb{P}_j(s) = \{s\}$. Consider the consumption bundle z which coincides with $\tilde{x}^j$ except for the commodity ℓ and the state s, where $z_\ell(s) = \tilde{x}_\ell^j(s) + \left(\sum_{i=1}^{N} \omega_\ell^i(s) - \sum_{i=1}^{N} \tilde{x}_\ell^i(s)\right)$. Observe that z is $\mathbb{P}_j$-measurable and since $\tilde{p}_\ell(s) = 0$, we have $\tilde{p}(s) \cdot z(s) = \tilde{p}(s) \cdot \tilde{x}(s)$. Therefore, (z, y) belongs to $B^j(\tilde{p}, \tilde{q})$ and by monotonicity of preferences, $U^j(z) > U^j(\tilde{x}^j)$, which is a contradiction.

Therefore, we conclude that, at equilibrium, commodities prices are strictly positive. Finally, the non-arbitrage condition allows us to ensure that asset prices are strictly positive as well.

Q.E.D.

Proof of Proposition 2.1. Without loss of generality, consider $p_1(s) = 1$ for every state s. Given the partition $\mathbb{P}_i$ for consumer i, let $\widetilde{\mathbb{P}}_i = \{P_1^i, ..., P_{K^i}^i\}$ be the corresponding set of elements of the partition such that $card(P_k^i) > 1$, for all $k = 1, ..., K^i$. For each k, fix a state of nature $\bar{s}_k$ in each element $P_k^i \in \widetilde{\mathbb{P}}_i$. Therefore, the information structure of the consumer i, leads to $\rho = \sum_{k=1}^{K_i}(card(P_k^i) - 1)$ restrictions over the consumption set which are given by the equations $x_{\bar{s}_k} = x(s)$, for every $s \in P_k^i$ such that $s \neq \bar{s}_k$ and for all $k = 1, ..., K^i$. For each $s \in P_k^i$, let γ_{ks} be the lagrange multiplier associated with the restriction $x(s) - x_{\bar{s}_k} = 0$. Let

μ be the lagrange multiplier associated with the budget constraint of the period 0 and let $\lambda_1, ..., \lambda_S$ denote the lagrange multipliers for the corresponding budget constraints of the period 1.

The lagrangian function for consumer i is given by:

$$\begin{aligned}\mathcal{L}^i(x_0, x_1, \lambda, y, \mu, \gamma) &= U^i(x_0, x_1) + \sum_{s=1}^{S} \lambda_s [R(s)y - p(s) \cdot (x(s) - w^i(s))] - \\ &\quad \mu[q \cdot y + p_0 \cdot (x_0 - w_0^i)] + \sum_{k=1}^{K^i} \sum_{\substack{s \in P_k^i \\ s \neq \bar{s}_k}} \gamma_{ks}(x_{\bar{s}_k} - x(s)).\end{aligned}$$

Note that the last term exhibits the restrictions of consumption required by agent i. Indeed, when $\mathbb{R}_+^{L(S+1)}$ is the consumption set for agent i this term is removed and in this case the differentiability property of the demand was already proved by Geanakoplos and Polemarchakis (1986).

The conditions which characterize the solution for the consumer problem are,

$$\begin{aligned}&D_0 U^i(x) - \mu p_0 = 0, \\ &D_s U^i(x) - \lambda_s p(s) + \Gamma_s = 0, s = 1, ..., S, \\ &R(s)y - p(s) \cdot (x(s) - w^i(s)) = 0, s = 1, ..., S, \\ &-\mu q + \lambda' R = 0, \\ &-p_0 \cdot (x_0 - w_0^i) - q \cdot y = 0, \\ &x_{\bar{s}_k} - x(s) = 0, \text{ for all } s \in P_k^i \text{ such that } s \neq \bar{s}_k, \text{ and for all } k = 1, ..., K^i,\end{aligned}$$

where $D_0 U^i$ and $D_s U^i$ denote the vector of partial derivatives of U^i with respect to $x_{0\ell}$ and $x_{s\ell}, \ell = 1, \ldots, L$, respectively; $\lambda' = (\lambda_1, \ldots, \lambda_S)$ and Γ_s is a vector with L coordinates such that for every $\ell = 1, ..., L$,

$$(\Gamma_s)_\ell = \begin{cases} 0 & \text{if } s \notin \bigcup_{k=1,\ldots,K^i} P_k^i, \\ \sum_{s' \in P_k^i, s' \neq \bar{s}_k} \gamma_{ks'} & \text{if } s \in P_k^i \text{ and } s = \bar{s}_k, \\ -\gamma_{ks} & \text{if } s \in P_k^i \text{ and } s \neq \bar{s}_k. \end{cases}$$

Let $\mathbf{J}$ be the Jacobian matrix of order $S(L+1) + S + B + 1 + \rho$ given by the second order derivatives with respect to $(x_0, x_1, \lambda, y, \mu, \gamma)$:

$$\mathbf{J} = \begin{bmatrix} D_0^2 U^i & 0 & 0 & 0 & -p_0 & 0 \\ 0 & D_1^2 U^i & -p_1 & 0 & 0 & V \\ 0 & -p_1' & 0 & R & 0 & 0 \\ 0 & 0 & R' & 0 & -q & 0 \\ -p_0 & 0 & 0 & -q' & 0 & 0 \\ 0 & V' & 0 & 0 & 0 & 0 \end{bmatrix}$$

where p_0 and p_1 denote the commodity price system of first and second period, respectively, and $V = \begin{bmatrix} V_1 \\ \vdots \\ V_S \end{bmatrix}$ is the matrix of order $SL \times \rho$ defined as follows:

$$\begin{aligned} V_{s\ell,ks'} = \frac{\partial(D_s U^i - \lambda_s p(s) + \Gamma_s)_\ell}{\partial \gamma_{ks'}} &= \frac{\partial(x_{\bar{s}_k \ell} - x_{s\ell})}{\partial \gamma_{ks'}} \\ &= \begin{cases} 0 & \text{if } s \notin \bigcup\limits_{k=1,\dots,K^i} P_k^i, \\ 1 & \text{if } s \in P_k^i \text{ and } s = \bar{s}_k, \\ 0 & \text{if } s \in P_k^i,\ s \neq \bar{s}_k \text{ and } s \neq s', \\ -1 & \text{if } s \in P_k^i,\ s \neq \bar{s}_k \text{ and } s = s'. \end{cases} \end{aligned}$$

It remains to prove that the matrix $\mathbf{J}$ is non-singular which, by applying the implicit function theorem, implies the continuous differentiability of the demand function. Let us show that if $\mathbf{J}z = 0$ then $z = 0$. For it, assume that $\mathbf{J}z = 0$, with $z = (\hat{x}_0, \hat{x}_1, \hat{\lambda}, \hat{y}, \hat{\mu}, \hat{\gamma})$ that is,

$$\begin{aligned} & D_0^2 U^i \hat{x}_0 - \hat{\mu} p_0 = 0 \\ & D_1^2 U^i \hat{x}_1 - p_1 \hat{\lambda} + V \hat{\gamma} = 0 \\ & -p_1' \hat{x}_1 + R \hat{y} = 0 \\ & R' \hat{\lambda} - \hat{\mu} q = 0 \\ & -p_0 \hat{x}_0 - q' \hat{y} = 0 \\ & V' \hat{x}_1 = 0. \end{aligned}$$

Then $z'\mathbf{J}z = 0$, using $\mathbf{J}z = 0$, reduces to $\hat{x}_0'(D_0^2 U^i)\hat{x}_0 + \hat{x}_1'(D_1^2 U^i)\hat{x}_1 = 0$. The negative definiteness of $D^2 U^i$ implies $\hat{x}_0 = 0$ and $\hat{x}_1 = 0$. Since $p_0 \neq 0$ the first equality guarantees $\hat{\mu} = 0$. By using the third equality and noticing that R has full colinear rank, we obtain $\hat{y} = 0$. Moreover, by using the second equality with $p(s) \neq 0$ and noticing that $V_s = 0$ for every state $s \notin \bigcup_{k=1}^{K^i} P_k^i$, it follows that $\hat{\lambda}_s = 0$, whenever s is one of the $S - B$ states for which agent i has no consumption restriction. Next, we use the equation $R'\hat{\lambda} - \hat{\mu}q = 0$ with $\hat{\mu} = 0$ and the fact that R is in general position to obtain $\hat{\lambda}_s = 0$ for the remaining states s, that is, for the states $s \in P^i_k$ for some $k = 1, \ldots, K^i$. Finally, the second equality with $\hat{x}_1 = 0$ and $\hat{\lambda} = 0$ allows us to conclude that $\hat{\gamma} = 0$.

Q.E.D.

Proof of Theorem 3.1. Given the economy $\mathcal{E}_I$ and a compact set $K \subset \mathbb{R}^{L(S+1)}_+ \times \mathbb{R}^B$, let us define a generalized game as in the proof of Theorem 2.1, with N players representing the consumers but with $C+1$ auctioneers instead of $S+1$[5]. Each consumer i maximizes U^i on the budget set $B^i(p,q) \bigcap K$ whereas the first period auctioneer chooses prices $(p_0, q) \in \Delta^{L+B-1}$ and maximizes $p_0 \cdot \sum_{i=1}^{N}(x^i_0 - \omega^i_0) + q \cdot \sum_{i=1}^{N} y^i$. Finally, each auctioneer e associated to an event $E_e \in \mathbb{P}$ chooses prices $p_e \in \Delta^{L-1}$ and maximize $\sum_{s \in E_e} p_e \cdot \sum_{i=1}^{N}(x^i(s) - w^i(s))$. We remark that the prices $p = (p_e, e = 1, \ldots, C) \in (\Delta^{L-1})^C$ selected by the C auctioneers are equivalent to a price system in $(\Delta^{L-1})^S$ which is $\mathbb{P}$-measurable, namely, $p(s) = p_e$ for every $s \in E_e$.

Note that the payoff functions for the C auctioneers in the second period are linear and, following the same argument as in the proof of Theorem 2.1, we conclude that there exists a sequence of equilibria (x^n, y^n, p^n, q^n) for the generalized games restricted to a sequence of increasing compact sets $K^n \subset \mathbb{R}^{L(S+1)}_+ \times \mathbb{R}^B$, such that $\left(2\sum_{i=1}^{N}\omega^i, 0\right) \in K^n$ for every n.

The behavior of the auctioneers associated to each common knowledge event implies that $\sum_{s \in E_e}\sum_{i=1}^{N}(x^{ni}(s) - w^i(s)) \leq 0$ for every E_e and, therefore, we conclude that x^{ni} belongs to the interior of K^n restricted to $\mathbb{R}^{L(S+1)}_+$, for every i. Assume that for some n there exists a state $\bar{s} \in E_e$ and a commodity ℓ such that $\sum_{i=1}^{N}(x^{ni}_\ell(\bar{s}) - w^i_\ell(\bar{s})) > 0$. Since x^{ni} belongs to the interior of $K^n \bigcap \mathbb{R}^{L(S+1)}_+$, as-

[5]Observe that the main difference between this generalized game and the previous one, in the proof of Theorem 2.1, is that here there are one auctioneer for the first period and one auctioneer for each common knowledge event of the second period instead of an auctioneer for each state of nature.

sumption $(A.5)$ holds which implies that $\sum_{i=1}^{N}(x_\ell^{in}(s)-w_\ell^i(s)) > 0$, for every $s \in E_e$, but this is in contradiction with the behavior of the auctioneer e corresponding to the common knowledge event E_e.

Thus, we conclude that there exists a subsequence converging to the limit (x^*, y^*, p^*, q^*). Note that since each p^n is $\mathbb{P}$-measurable, the limit price p^* is $\mathbb{P}$-measurable as well and, then p^* does not reveal any information. To finish the proof, we will show that (x^*, y^*, p^*, q^*) is an equilibrium for the economy $\mathcal{E}_I$.

Since R has rank B, arguing again as in the proof of Theorem 2.1, we can deduce that x^{*i} is $\mathbb{P}_i$-measurable and (x^{*i}, y^{*i}) is an optimal choice for consumer i at prices (p^*, q^*). Moreover, the auctioneer in the first period guarantees that financial and spot markets at $t = 0$ clear, and the auctioneer e for every common knowledge event guarantees that there is no excess of supply in spot markets. Finally, monotonicity of preferences and assumption (A.3) implies that spot markets clear in the second period.

Q.E.D.

Proof of Theorem 4.1 Let $\Lambda(\cdot)$ be an operator that maps a vector $\lambda = (\lambda_e, e = 1, \ldots, C) \in \mathbb{R}_+^C$ into a diagonal matrix $\Lambda(\lambda)$ of order $S \times S$ whose diagonal elements are given by $\lambda(s) = \lambda_e$ for every $s \in E_e$.

Consider a non-informational equilibrium $(p, q), (x, y)$ for the economy $\mathcal{E}_I$ where the returns of nominal assets are given by the matrix R. Note that since p is $\mathbb{P}$-measurable, we can write $p(s) = p_e$ for every $s \in E_e$ and each $e = 1, \ldots, C$. Then, it is easy to check that $(p, q), (x, y)$ is also a real numeraire asset equilibrium with commodity 1 return matrix $\Lambda(\lambda) \cdot R$, with $\lambda_e = \dfrac{1}{p_{e1}}$. Thus, the set of non-informational equilibria is included in the set of real numeraire equilibria associated to return matrix $\Lambda(\lambda) \cdot R$ for the different vectors $\lambda \in \mathbb{R}_+^C$. Further, the equilibrium with real numeraire assets is determinate and when $\lambda = \alpha\lambda'$ for some $\alpha > 0$, the subspaces generated by $\Lambda(\lambda) \cdot R$ and $\Lambda(\lambda') \cdot R$ are the same and, then, the corresponding equilibrium allocations coincide.

Therefore, we conclude that the dimension of real indeterminacy of the set of non-informational equilibria is at most $C - 1$.

Q.E.D.

Proof of Theorem 4.2. Let $p \gg 0$ be a commodity price system that is $\mathbb{P}$-measurable. Then, we can consider that (p, q) belongs to $\mathbb{R}_{++}^{(L-1)(C+1)+B}$.

Since ω^i is $\mathbb{P}_i$-measurable for every consumer i, and there exists a set $\mathcal{N}$ of $B+1$ consumers such that $\mathbb{P}_i = \mathbb{P}$ for every $i \in \mathcal{N}$, we can also consider $\omega = (\omega^i, i = 1, \ldots, N) \in \mathbb{R}^{\mathcal{H}}_{++} \times \mathbb{R}^{(B+1)L(C+1)}_{++}$, where $\mathcal{H} = \sum_{i \notin \mathcal{N}} L(k_i + 1)$ and k_i is the number of events in $\mathbb{P}_i$.

Let us fix $\omega_{N \setminus \mathcal{N}} \in \mathbb{R}^{\mathcal{H}}_{++}$, i.e., the initial endowments for agents which are not in $\mathcal{N}$. Given $(p, q, \lambda, \omega_{\mathcal{N}}) \in \mathcal{M} = \mathbb{R}^{(L-1)(C+1)}_{++} \times \mathbb{R}^B_+ \times \mathbb{R}^C_+ \times \mathbb{R}^{(B+1)L(C+1)}_{++}$, let $f(p, q, \lambda, \omega_{\mathcal{N}})$ denote the excess demand function for assets and commodities other than the numeraire for the economy with return matrix $D(\lambda)R$, where $D(\lambda)$ is the diagonal matrix of order $S \times S$ which is $\mathbb{P}$-measurable and is defined by λ[6]. Since $\mathbb{P}_i = \mathbb{P}$ for every $i \in \mathcal{N}$, by assumption (A.5), we can conclude that $f(p, q, \lambda, \omega_{\mathcal{N}})$ is $\mathbb{P}$-measurable for every $\omega_{\mathcal{N}} \in \mathbb{R}^{(B+1)L(C+1)}_{++}$ and then we can consider that f takes values on $\mathbb{R}^{(L-1)(C+1)}_{++} \times \mathbb{R}^B_+$.[7]

Since R_C has full rank and (A.1)' holds, we can deduce that f is a continuously differentiable function. Furthermore, $f(p, q, \lambda, \omega_{\mathcal{N}}) = 0$ implies that $rank\ \partial_{\omega^{\bar{i}}} f(p, q, \lambda, \omega_{\mathcal{N}}) = (L-1)(C+1) + B$, being $\bar{i}$ a consumer in $\mathcal{N}$ (see Geanakoplos and Polemarchakis (1986) for details), that is, f is transverse to 0.

Now, to prove (i), let z an element in the $B-1$ sphere, denoted by $\mathcal{Z}$, and define

$$g(p, q, \lambda, \omega_{\mathcal{N}}, z) = \left(f(p, q, \lambda, \omega_{\mathcal{N}}), \sum_{h=1}^{B} z_h y_1^h, \ldots, \sum_{h=1}^{B} z_h y_B^h \right),$$

where y_b^h is the demand for asset b at $(p, q, \lambda, \omega_{\mathcal{N}})$ for consumer $h \in \mathcal{N} \setminus \{\bar{i}\}$. If $g(p, q, \lambda, \omega_{\mathcal{N}}, z) = 0$, then $\partial_{\omega_{\mathcal{N}}} g(p, q, \lambda, \omega_{\mathcal{N}}, z) = (L-1)(C+1) + 2B$ (to see this, follow the proof of Lemma 3 in Geanakoplos and Mas-Colell (1989) taking into account that R and p are $\mathbb{P}$-measurable).

Since $\mathbb{P}_i = \mathbb{P}$ for every $i \in \mathcal{N}$, we have that $f^{-1}_{\omega_{\mathcal{N}}}(0)$ and $g^{-1}_{\omega_{\mathcal{N}}}(0)$ are non-empty. Then, by the Transversality Theorem (see, for instance, Mas-Colell (1985), subsection 1.I), for almost all $\omega_{\mathcal{N}}$, the sets $f^{-1}_{\omega_{\mathcal{N}}}(0)$ and $g^{-1}_{\omega_{\mathcal{N}}}(0)$ are C^1 manifolds of dimension C and $C-1$ respectively. By Sard's theorem (see also Mas-Colell (1985)) the projection of $f^{-1}_{\omega_{\mathcal{N}}}(0)$ on the set $\mathcal{D}$ (where $\mathcal{D}$ denotes the set of diagonal positive matrixes of order $S \times S$ which are $\mathbb{P}$-measurable) has a regular value $\widehat{\lambda} \in \mathbb{R}^C_{++}$.

[6]Let $\lambda = (\lambda_1, \ldots, \lambda_C)$ and $\mathbb{P} = \{P_1, P_2, \ldots, P_C\}$. Then, for each state $s \in P_h$ the s diagonal entry of $D(\lambda)$ is λ_h.

[7]The $S+1$ occurrences of the Walras law allows us to drop S+1 markets; thus, if that markets are precisely the market of the numeraire in the first period and the market of numeraire in each state of the second period, the aggregate excess demand function in the remaining markets take values on $\mathbb{R}^{(L-1)(S+1)}$.

Applying Theorem 3.1, we can conclude that there exists a non-informational equilibrium price system $(\widehat{p},\widehat{q})$ such that $(\widehat{p},\widehat{q},\widehat{\lambda},\omega_{\mathcal{N}}) \in f^{-1}(0)$, that is, $\widehat{\lambda}$ is actually in the range of the projection. Then, by the Implicit Function Theorem, there are open sets $M \subset \mathcal{M}_{\omega_{\mathcal{N}}} = \{(p,q,\lambda) : \ (p,q,\lambda,\omega_{\mathcal{N}}) \in \mathcal{M}\}$, $A \subset \mathbb{R}^C_{++}$, and a C^1 function $\xi : A \to M$ such that $(p,q,\lambda,\omega_{\mathcal{N}}) \in f^{-1}(0) \cap (\mathcal{M}_{\omega_{\mathcal{N}}} \times \{\omega_{\mathcal{N}}\})$ if and only if $\xi(\lambda) = (p,q,\lambda)$. Let $\overline{M} \subset \mathcal{M}_{\omega_{\mathcal{N}}}$ the closure of M. Then, the projection of $g^{-1}_{\omega_{\mathcal{N}}}(0) \cap (\overline{M} \times \mathcal{Z})$ on $\mathcal{Z}$ is compact and therefore there is an open set $D \subset A$ which is disjoint from this projection. This means that if $\lambda \in D$ then the individual assets demands, corresponding to $\xi(\lambda)$ span $\mathbb{R}^B$.

On the other hand, since $B < C$ and R_C is in general position, by Lemma 4 in Geanakoplos and Mas-Colell (1989), we have that $sp[D(\lambda)R] = sp[D(\lambda')R]$ if and only if $\lambda = \alpha\lambda'$ for some $\alpha > 0$. To complete the proof of (i) it remains to apply Lemma 1 in the cited paper by Geanakoplos and Mas-Colell.

Finally, to proof (ii), let J denotes the $C(B-1)$ sphere and take a set $\mathcal{N}$ of $CB+1$ consumers such that $\mathbb{P}_i = \mathbb{P}$ for every $i \in \mathcal{N}$ and let us define a function G from $\mathcal{M} \times J$ to $\mathbb{R}^{(L-1)(C+1)} \times \mathbb{R}^B \times \mathbb{R}^{CB}$, where $G(p,q,\lambda,\omega_{\mathcal{N}},z^1,\ldots,z^C)$ is given by

$$\left(f(p,q,\lambda,\omega_{\mathcal{N}}), \sum_{h=1}^{B} z^1_h y^h_1, \ldots, \sum_{h=1}^{B} z^1_h y^h_B, \ldots, \sum_{h=1}^{B} z^C_h y_1^{(C-1)B+h}, \ldots, \sum_{h=1}^{B} z^C_h y_B^{(C-1)B+h} \right),$$

where y^k_b is the demand for asset b at $(p,q,\lambda,\omega_{\mathcal{N}})$ for consumer $k \in \mathcal{N} \setminus \{\bar{i}\}$.

Note that the range of $G_{\omega_{\mathcal{N}}}$ has greater dimension than its domain. Then, reasoning as in the proof of (i), we can conclude that, generically, $G^{-1}_{\omega_{\mathcal{N}}}(0) = \emptyset$ and then $f^{-1}_{\omega_{\mathcal{N}}}(0)$ is a C manifold. To finish the proof, we can follow the same argument as in Remark 5 in Geanakoplos and Mas-Colell in order to conclude that $P = \{(p,q,\lambda,\omega_{\mathcal{N}} \in f^{-1}_{\omega_{\mathcal{N}}}(0) : \ \lambda_1 = 0\}$ is a $C-1$ manifold and the real allocations corresponding to any two points in P are necessarily distinct.

Q.E.D.

References

Balasko, Y., Cass, D., (1989): The Structure of Financial Equilibrium with Exogenous Yields: The Case of Incomplete Markets. Econometrica 57, 135-162.

Bisin, A. (1998): General Equilibrium with Endogenously Incomplete Financial Markets. Journal of Economic Theory 82(1), 19-45.

Cornet, B., De Boisdeffre L. (2002): Arbitrage and Price Revelation with Asymmetric Information and Incomplete Markets. Journal of Mathematical Economics, 38, 393-410

Faias, M., Moreno-García, E., Páscoa, M. (2002): Real Indeterminacy of Equilibria and Manipulability. Journal of Mathematical Economics 37, 325-340.

Geanakoplos, J., Mas-Colell, A. (1989): Real indeterminacy with financial assets. Journal of Economic Theory 47, 2238.

Geanakoplos, J., Polemarchakis, H.: Existence, Regularity, and Constrained Suboptimality of Competitive Allocations when the Asset Market Is Incomplete. In W. Heller, R. Starr, and D. Starrett (eds.), Essays in Honor of Kenneth Arrow, Vol. 3. Cambridge University Press, 1986, pp. 65-95.

Magill, M., Quinzi, M. (1992): Real Effects of Money in General Equilibrium. Journal of Mathematical Economics 21, 301-342.

Mas-Colell, A. (1985): The Theory of General Economic Equilibrium: A Differentiable Approach. Cambridge University Press, Cambridge.

Pesendorfer, W. (1995): Financial Innovation in a General Equilibrium Model. Journal of Economic Theory 65(1), 79-116.

Polemarchakis, H., Siconolfi P. (1993): Asset Markets and the Information Revealed by Prices. Economic Theory 3, 645-661.

Radner, R. (1968): Competitive Equilibrium under Uncertainty. Econometrica 36, 31-58.

Radner, R. (1979): Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices . Econometrica 47(3), 655-678.

Rahi, R. (1995): Partially Revealing Rational Expectations Equilibria with Nominal Assets. Journal of Mathematical Economics 24, 137-146.

Werner, J. (1985): Equilibrium in Economies with Incomplete Financial Markets. Journal of Economic Theory 36, 110-119.

Yannelis, N. (1991): The Core of an Economy with Differential Information. Economic Theory 1, 183-198.